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SHORTER COMMUNICATIONS

INFLUENCE ON THERMAL DIFFUSION OF THE INCLINATION OF A COLUMN WITH BOTH ENDS CLOSED

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NOMENCLATURE

a, distance between the walls;
C, constant, $\dfrac{\nu^3}{2C_p a^2 \chi \delta T}$;
C_p, specific heat at constant pressure;
D, ordinary diffusion coefficient;
D', thermal diffusion coefficient;
g, gravitational acceleration;
G, Grashof number, $g\dfrac{\alpha a^3 \delta T}{\nu^2}$;
h, height of the column;
$\mathbf{J}$, solute flux;
N, mass fraction of solute;
P, pression in the fluid;
Pr, Prandtl number, $\dfrac{\nu}{\chi}$;
R, thermodiffusion Rayleigh number, $\dfrac{g\gamma a^3 N_0}{\nu^2}$;
Sc, Schmidt number, $\dfrac{\nu}{D}$;
T_1, temperature of the cold wall;
T_2, temperature of the hot wall, $T_1+\delta T$;
$\mathbf{V}$, convection velocity;
V_x, V_z, components of $\mathbf{V}$;
x, z, coordinates defined in Fig. 1.

Greek symbols

α, thermal expansion coefficient, $-\dfrac{1}{\rho_0}\left(\dfrac{\delta\rho}{\delta T}\right)_{N_0}$;
γ, coefficient of density variation due to mixture composition, $\dfrac{1}{\rho_0}\left(\dfrac{\delta\rho}{\delta N}\right)_{T_0}$;
δ, angle of inclination from the horizontal;
ν, kinematic viscosity;
ρ, density;
σ, thermodiffusion coefficient, $\dfrac{D'}{D}\delta T$;
φ, vorticity;
χ, thermal diffusivity;
Ψ, stream function.

Subscript
0, reference state.

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INTRODUCTION

Proposed in 1938 by Clusius and Dickel [1] to separate effectively the components of a binary mixture of gases, the thermogravitational effect results from the coupling of thermal diffusion, also called Soret effect, and of convective currents of a fluid in the gravitationnal field. The first theory of thermogravitation was given by Furry and Jones [2] for gases and later by De Groot [3] for dilute binary mixture of liquids. In a flat plate column the phenomenon can be described as follows: a horizontal temperature gradient applied to the initially homogeneous fluid gives rise to a horizontal concentration gradient (thermal diffusion). When the liquid mixture has a positive Soret coefficient, the two colinear gradients are in opposite directions; in the gravitational field, convective currents arise, downwards near the cold wall, upward near the hot wall. The convection brings, therefore more concentrated fluid towards the bottom of the column and when the steady state is reached a vertical concentration gradient is built between the top and the bottom of the apparatus. However, if convection is necessary to produce the desirable cascading effect, it has also an undesirable remixing effect in the fluid particularly if the convective strength is important. Then, to obtain vertical concentration gradients as high as possible, the convection must be preserved but controlled.

One way to adjust convection is to use packed [4] or wired [5] thermogravitational columns. Another way consists in tilting a flat plate column with hot wall on top to reduce the effective gravitational force.

Powers and Wilke [6] have given an expression for the separation in an inclined column, working in continuous flow, with binary feed introduced in the middle of the column and top and bottom products withdrawn at the same rates. In that case, the theoretical values of the constants representing the separation effectiveness by thermal diffusion and the effect of remixing by convection and by ordinary diffusion in the vertical direction, rarely agree with experiments; they must be corrected by empirical relations which allow the vertical concentration gradient to vary with horizontal coordinates [7].

Using Powers and Wilke theory, Chue and Yeh [8] discussed the conditions for the best performance of a flat plate continuous flow column. Later Sanchez [9] showed that the value of the constant characterizing the separation by thermal diffusion could not be the same for a continuous flow column and for a closed end one.

The purpose of this work is to determine if the separation is improved by tilting a column closed at both ends. We present here the results obtained by solving numerically the equations of the thermogravitation in the case of a dilute binary solution contained in an inclined both ends closed column when the steady state is reached.

FORMULATION OF THE PROBLEM

The model described here is a two dimensional one. The column (Fig. 1) is closed at the top and at the bottom by two adiabatic walls.

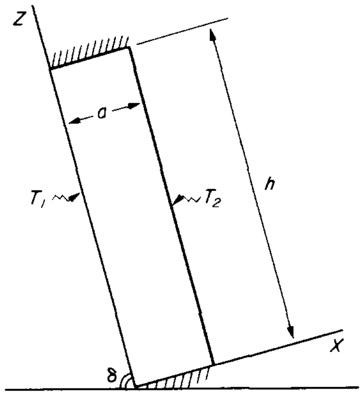

Fig. 1. Scheme of the inclined column with both ends closed.

Assuming that the convective flow is free from turbulence and that the velocity has two components, we study the stationary state of the column filled with a binary dilute solution. With Boussinesq approximation, the starting equations expressed the conservation of energy, mass and momentum:

$$\mathbf{V}\,.\,\nabla T = \chi\Delta T + \frac{\nu}{2C_p}\left[\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x}\right]^2; \quad (1)$$

$$\nabla\,.\,\mathbf{J} = 0 \text{ where } \mathbf{J} = -\rho D\nabla N - \rho D'\nabla T + \rho \mathbf{V} N; \quad (2)$$

$$(\mathbf{V}\,.\,\nabla)\,\mathbf{V} = -\frac{1}{\rho_0}\nabla P + \nu\Delta\mathbf{V} + \frac{\rho}{\rho_0}\boldsymbol{g}, \quad (3)$$

with the incompressibility condition

$$\nabla\,.\,\mathbf{V} = 0. \quad (4)$$

In addition the state equation which allows the density to depend upon temperature and concentration is:

$$\rho = \rho_0[1 - \alpha(T - T_0) + \gamma(N - N_0)]. \quad (5)$$

Taking into account the edge effects of the column, those equations have been discussed and solved in reference [10] for the case of a vertical flat plate column. When the column is tilted, the projection of the buoyancy term in the momentum equation are $-\rho g \cos\delta$ along $0x$ and $-\rho g \sin\delta$ along $0z$.

Introducing as usual the stream function Ψ:

$$V_x = \frac{\partial\Psi}{\partial z}, \qquad V_z = -\frac{\partial\Psi}{\partial x},$$

and the vorticity $\varphi = \Delta\Psi$, the system (1)–(4) can be reduced to the following where the unknowns are represented now in dimensionless form:

$$Pr\left[\frac{\partial T}{\partial x}\frac{\partial\Psi}{\partial z} - \frac{\partial T}{\partial z}\frac{\partial\Psi}{\partial x}\right] = \Delta T + C\left[\frac{\partial^2\Psi}{\partial x^2} - \frac{\partial^2\Psi}{\partial z^2}\right]^2; \quad (6)$$

$$\Delta N + \sigma N\delta T + (\sigma - \alpha\delta T)\left[\frac{\partial N}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial N}{\partial z}\frac{\partial T}{\partial z}\right] + Sc\alpha N\delta T\left[\frac{\partial\Psi}{\partial z}\frac{\partial T}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial T}{\partial z}\right] + \gamma\left[\left(\frac{\partial N}{\partial x}\right)^2 + \left(\frac{\partial N}{\partial z}\right)^2\right] - N\sigma\alpha\delta T\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right] - Sc\left[\frac{\partial\Psi}{\partial z}\frac{\partial N}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial N}{\partial z}\right] = 0 \quad (7)$$

$$\frac{\partial\varphi}{\partial x}\frac{\partial\Psi}{\partial z} - \frac{\partial\varphi}{\partial z}\frac{\partial\Psi}{\partial x} = \Delta\varphi - G\left[\frac{\partial T}{\partial x}\sin\delta - \frac{\partial T}{\partial z}\cos\delta\right] - R\left[\frac{\partial N}{\partial x}\sin\delta - \frac{\partial N}{\partial z}\cos\delta\right] = 0. \quad (8)$$

The scale factors for distance, temperature, mass fraction are respectively a, a/ν, δT, and N_0, which is the mass fraction of the initially homogeneous solution.

Equations (6)–(8) are subjected to boundary conditions on rigid impermeable walls: all components of velocity vanish at the boundaries; the temperatures of the side walls are imposed and the walls which close the column on top and on the bottom are insulated.

RESULTS

The derivatives of equations (6)–(8) were approximated by finite differences and the system was then solved at discrete intervals of space. The method of successive iterations with an over relaxation factor has been used as computational procedure [11] on a CDC 6600 computer. Each iteration needs about one second. The initial values were those computed for a dilute solution of ClNa in water (0.1 $\mathrm{mol\,l^{-1}}$) contained in a column of 1 mm width and 40 cm height when it is vertical ($\delta = 90°$). The temperature difference imposed on the fluid is $\delta T = 10°C$.

The numerical values of the fluid parameters used here are:

$Pr = 7$, $\alpha = 2.5\,10^{-4}\,°C^{-1}$, $\gamma = 18.2\,\mathrm{l\,mol^{-1}}$,
$\chi = 14.5\,10^{-4}\,\mathrm{cm^2\,s^{-1}}$, $\nu = 10^{-2}\,\mathrm{cm^2\,s^{-1}}$,
$D = 1.4\,10^{-5}\,\mathrm{cm^2\,s^{-1}}$, $\frac{D'}{D} = 2.3\,10^{-3}\,°C^{-1}$.

In that case we have shown [10] that there is a unique convection roll in the fluid and that the effects of the ends of the column are appreciable on about 2.5 cm from the top and bottom. So we have computed for different values of the angle δ, decreasing from 90° by steps of 2°, the average values of the concentration at the top CT and at the bottom CB on a height of 5 cm that is twice the edge effects.

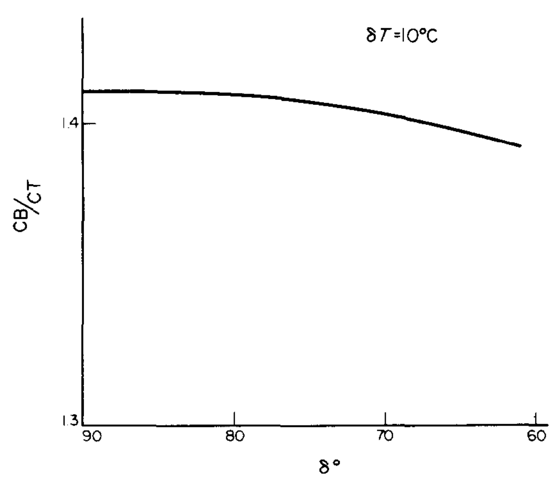

Fig. 2. Effect of the column inclination on ratio CB/CT, with a unique convective roll.

Figure 2 shows the variation of CB/CT when δ varies from 90° to 60°. It can be seen first that a strict verticality of the apparatus is useless to obtain the best performance: the ratio CB/CT is quite constant when δ decreases between 90° and 80°. Moreover, the inclination of a closed end column does not seem to improve the expected separation.

CONCLUSION

The results of this theoretical study are to be seen in an experimental perspective:

(1) The fact that the verticality of the column is not important is of practical interest in an experiment, as measurements of Soret coefficients of dilute binary solutions which can be done, using a simple method [12] with a thermogravitational column closed at both ends. In such an experiment an imperfection of the verticality of the apparatus is not prejudicial to the measurements and moreover, the apparatus is lighter and less expensive than one using a continuous flow column because it does not need a feed system to provide steady flow through the column.

(2) In the case of a flat plate column closed at both ends, the inclination of the apparatus does not seem to improve the expected separation. In the state of our results we do not know the variation of CB/CT for angles δ smaller than 60° but what we can say is that the more tilted the column is the more it approaches an horizontal thermodiffusion cell for which there is no appreciable longitudinal separation.

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EXTENSION OF THE SOLUTION OF INVERSE CONDUCTION PROBLEM

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NOMENCLATURE

b,	slab thickness;
Bi,	Biot number, hb/K;
h,	convective heat transfer coefficient;
k_0,	reference thermal conductivity at $T = T_0$;
t,	nondimensional time, $(\alpha_0\tau)/b^2$;
T,	temperature;
T_g,	driving gas temperature;
T_0,	initial temperature;
X,	dimensionless coordinate, x/b;
x,	space coordinate.

Greek symbols

α_0,	reference thermal diffusivity;
β,	constant (thermal conductivity coefficient);
τ,	time;
θ,	dimensionless temperature, $(T-T_0)/(T_g-T_0)$.

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INTRODUCTION

THE HIGH temperature range involved and the considerable variation of thermal conductivity with temperature for many present-day materials require that the variation of conductivity with temperature be considered in the analysis of the inverse conduction problem. In a previous study [1], a minimization technique was developed for the estimation of convective heat transfer coefficient and wall temperature from experimental temperature data. The current investigation represents the extension of this previously developed method by taking into consideration temperature-dependent thermal conductivity.

Beck [2] used a finite-difference approximation in conjunction with least-square fit procedure as well as nonlinear estimate method to solve the inverse conduction problem. Howard [3] developed a numerical procedure for determining the heat flux to a thermally thick wall with variable thermal properties using a single embedded thermocouple. His best results were obtained for temperature measurements close to the heated surface in con-